Graphs Using Slope-Intercept Form

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Graphs Using Slope-Intercept Form

Here you’ll learn how to identify the slope and \( y \)-intercept of equations. You’ll also learn how to graph equations that are in slope-intercept form.

What if you were given the equation of a line like \( y = -2x - 7 \)? How could you determine its slope and \( y \)-intercept? After completing this Concept, you’ll be able to identify the slope and \( y \)-intercept of linear equations like this one.

**Watch This**

[CK-12 Foundation: 0408S Graphs Using Slope-Intercept Form (H264)]

**Try This**

To get a better understanding of what happens when you change the slope or the \( y \)-intercept of a linear equation, try playing with the Java applet at http://standards.nctm.org/document/eexamples/chap7/7.5/index.htm.

**Guidance**

The total profit of a business is described by the equation \( y = 15000x - 80000 \), where \( x \) is the number of months the business has been running. How much profit is the business making per month, and what were its start-up costs? How much profit will it have made in a year?

**Identify Slope and \( y \)-intercept**

So far, we’ve been writing a lot of our equations in **slope-intercept form**—that is, we’ve been writing them in the form \( y = mx + b \), where \( m \) and \( b \) are both constants. It just so happens that \( m \) is the slope and the point \((0, b)\) is the \( y \)-intercept of the graph of the equation, which gives us enough information to draw the graph quickly.

**Example A**

*Identify the slope and \( y \)-intercept of the following equations.*

a) \( y = 3x + 2 \)

b) \( y = 0.5x - 3 \)

c) \( y = -7x \)

d) \( y = -4 \)

**Solution**

a) Comparing
, we can see that \( m = 3 \) and \( b = 2 \). So \( y = 3x + 2 \) has a **slope of 3** and a **y−intercept of (0, 2)**.

b) \( y = 0.5x - 3 \) has a **slope of 0.5** and a **y−intercept of (0, -3)**.

Notice that the intercept is **negative**. The \( b \)-term includes the sign of the operator (plus or minus) in front of the number—for example, \( y = 0.5x - 3 \) is identical to \( y = 0.5x + (-3) \), and that means that \( b \) is -3, not just 3.

c) At first glance, this equation doesn’t look like it’s in slope-intercept form. But we can rewrite it as \( y = -7x + 0 \), and that means it has a **slope of -7** and a **y−intercept of (0, 0)**. Notice that the slope is negative and the line passes through the origin.

d) We can rewrite this one as \( y = 0x - 4 \), giving us a **slope of 0** and a **y−intercept of (0, -4)**. This is a horizontal line.

**Example B**

*Identify the slope and y−intercept of the lines on the graph shown below.*

The intercepts have been marked, as well as some convenient lattice points that the lines pass through.

**Solution**

a) **The y−intercept is (0, 5)**. The line also passes through (2, 3), so the slope is \( \frac{\Delta y}{\Delta x} = \frac{-2}{2} = -1 \).

b) **The y−intercept is (0, 2)**. The line also passes through (1, 5), so the slope is \( \frac{\Delta y}{\Delta x} = \frac{3}{1} = 3 \).

c) **The y−intercept is (0, -1)**. The line also passes through (2, 3), so the slope is \( \frac{\Delta y}{\Delta x} = \frac{4}{2} = 2 \).

d) **The y−intercept is (0, -3)**. The line also passes through (4, -4), so the slope is \( \frac{\Delta y}{\Delta x} = \frac{-1}{4} = -\frac{1}{4} \) or -0.25.
Graph an Equation in Slope-Intercept Form

Once we know the slope and intercept of a line, it’s easy to graph it. Just remember what slope means. Let’s look back at this example from Lesson 4.1.

Ali is trying to work out a trick that his friend showed him. His friend started by asking him to think of a number, then double it, then add five to the result. Ali has written down a rule to describe the first part of the trick. He is using the letter x to stand for the number he thought of and the letter y to represent the final result of applying the rule. He wrote his rule in the form of an equation: \( y = 2x + 5 \).

Help him visualize what is going on by graphing the function that this rule describes.

In that example, we constructed a table of values, and used that table to plot some points to create our graph.

We also saw another way to graph this equation. Just by looking at the equation, we could see that the \( y \)-intercept was (0, 5), so we could start by plotting that point. Then we could also see that the slope was 2, so we could find another point on the graph by going over 1 unit and up 2 units. The graph would then be the line between those two points.

Here’s another problem where we can use the same method.

**Example C**

*Graph the following function: \( y = -3x + 5 \)*

**Solution**

To graph the function without making a table, follow these steps:
1. Identify the y-intercept: \( b = 5 \)
2. Plot the intercept: (0, 5)
3. Identify the slope: \( m = -3 \). (This is equal to \( -\frac{3}{1} \), so the rise is -3 and the run is 1.)
4. Move over 1 unit and down 3 units to find another point on the line: (1, 2)
5. Draw the line through the points (0, 5) and (1, 2).

Notice that to graph this equation based on its slope, we had to find the rise and run—and it was easiest to do that when the slope was expressed as a fraction. That’s true in general: to graph a line with a particular slope, it’s easiest to first express the slope as a fraction in simplest form, and then read off the numerator and the denominator of the fraction to get the rise and run of the graph.

**Example D**

Find integer values for the rise and run of the following slopes, then graph lines with corresponding slopes.

a) \( m = 3 \)

b) \( m = -2 \)

**Solution**

a)

\[ 3 = \frac{3}{1} \quad \text{As we move across 1 unit we move up by 3} \]

b)
Changing the Slope or Intercept of a Line

The following graph shows a number of lines with different slopes, but all with the same y-intercept: (0, 3).

You can see that all the functions with positive slopes increase as we move from left to right, while all functions with negative slopes decrease as we move from left to right. Another thing to notice is that the greater the slope, the steeper the graph.

This graph shows a number of lines with the same slope, but different y-intercepts.

Notice that changing the intercept simply translates (shifts) the graph up or down. Take a point on the graph of \( y = 2x \), such as (1, 2). The corresponding point on \( y = 2x + 3 \) would be (1, 5). Adding 3 to the y-intercept means...
we also add 3 to every other $y-$value on the graph. Similarly, the corresponding point on the $y = 2x - 3$ line would be $(1, -1)$; we would subtract 3 from the $y-$value and from every other $y-$value.

Notice also that these lines all appear to be parallel. Are they truly parallel?

To answer that question, we’ll use a technique that you’ll learn more about in a later chapter. We’ll take 2 of the equations—say, $y = 2x$ and $y = 2x + 3$—and solve for values of $x$ and $y$ that satisfy both equations. That will tell us at what point those two lines intersect, if any. (Remember that parallel lines, by definition, are lines that don’t intersect.)

So what values would satisfy both $y = 2x$ and $y = 2x + 3$? Well, if both of those equations were true, then $y$ would be equal to both $2x$ and $2x + 3$, which means those two expressions would also be equal to each other. So we can get our answer by solving the equation $2x = 2x + 3$.

But what happens when we try to solve that equation? If we subtract $2x$ from both sides, we end up with $0 = 3$. That can’t be true no matter what $x$ equals. And that means that there just isn’t any value for $x$ that will make both of the equations we started out with true. In other words, there isn’t any point where those two lines intersect. They are parallel, just as we thought.

And we’d find out the same thing no matter which two lines we’d chosen. In general, since changing the intercept of a line just results in shifting the graph up or down, the new line will always be parallel to the old line as long as the slope stays the same.

Any two lines with identical slopes are parallel.

Watch this video for help with the Examples above.

CK-12 Foundation: Graphs Using Slope-Intercept Form

Vocabulary

- A common form of a line (linear equation) is slope-intercept form: $y = mx + b$, where $m$ is the slope and the point $(0, b)$ is the $y-$intercept
- Graphing a line in slope-intercept form is a matter of first plotting the $y-$intercept $(0, b)$, then finding a second point based on the slope, and using those two points to graph the line.
- Any two lines with identical slopes are parallel.

Guided Practice

Find integer values for the rise and run of the following slopes, then graph lines with corresponding slopes.

a) $m = 0.75$

b) $m = -0.375$

Solution:

a)
0.75 = \frac{3}{4}. As we move across 4 units we move up by 3.

-0.375 = -\frac{3}{8}. As we move across 8 units we move down by 3.

b)

Practice

Identify the slope and $y$—intercept for the following equations.

1. $y = 2x + 5$
2. $y = -0.2x + 7$
3. $y = x$
4. $y = 3.75$

Identify the slope of the following lines.

Identify the slope and $y$—intercept for the following functions.
For 7-10, plot the following functions on a graph.

7. \( y = 2x + 5 \)
8. \( y = -0.2x + 7 \)
9. \( y = x \)
10. \( y = 3.75 \)

11. Which two of the following lines are parallel?
   a. \( y = 2x + 5 \)
   b. \( y = -0.2x + 7 \)
   c. \( y = x \)
   d. \( y = 3.75 \)
   e. \( y = -\frac{1}{3}x - 11 \)
   f. \( y = -5x + 5 \)
   g. \( y = -3x + 11 \)
   h. \( y = 3x + 3.5 \)

12. What is the \( y \)-intercept of the line passing through \((1, -4)\) and \((3, 2)\)?
13. What is the \( y \)-intercept of the line with slope -2 that passes through \((3, 1)\)?
14. Line \( A \) passes through the points \((2, 6)\) and \((-4, 3)\). Line \( B \) passes through the point \((3, 2.5)\), and is parallel to line \( A \)
   a. Write an equation for line \( A \) in slope-intercept form.
   b. Write an equation for line \( B \) in slope-intercept form.
15. Line \( C \) passes through the points \((2, 5)\) and \((1, 3.5)\). Line \( D \) is parallel to line \( C \), and passes through the point \((2, 6)\). Name another point on line \( D \). (Hint: you can do this without graphing or finding an equation for either line.)